# Transients and Oscillations in RLC Circuits

## Physics 401, Spring 2013.

*Eugene V. Colla*

# Transients and Oscillations in RLC Circuits. Outline

Transients. Definition.

- Transients in RLC
- Resonance in RLC

Data analysis. Origin. Fitting.

Some complimentary but very important stuff:

Voltage source, current source,

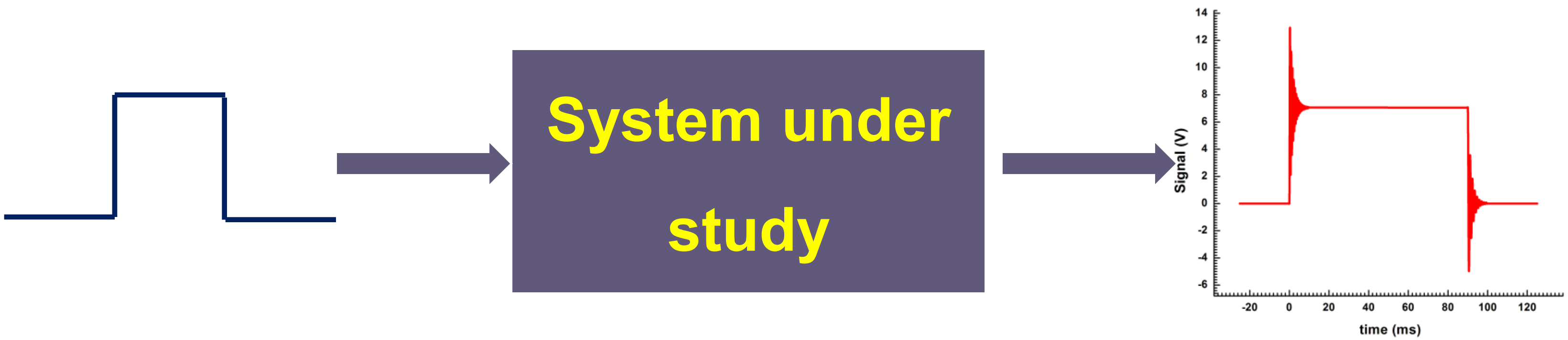
Input resistance, output resistance

Two probe, four probe measuring technique

# Transients. Definition.

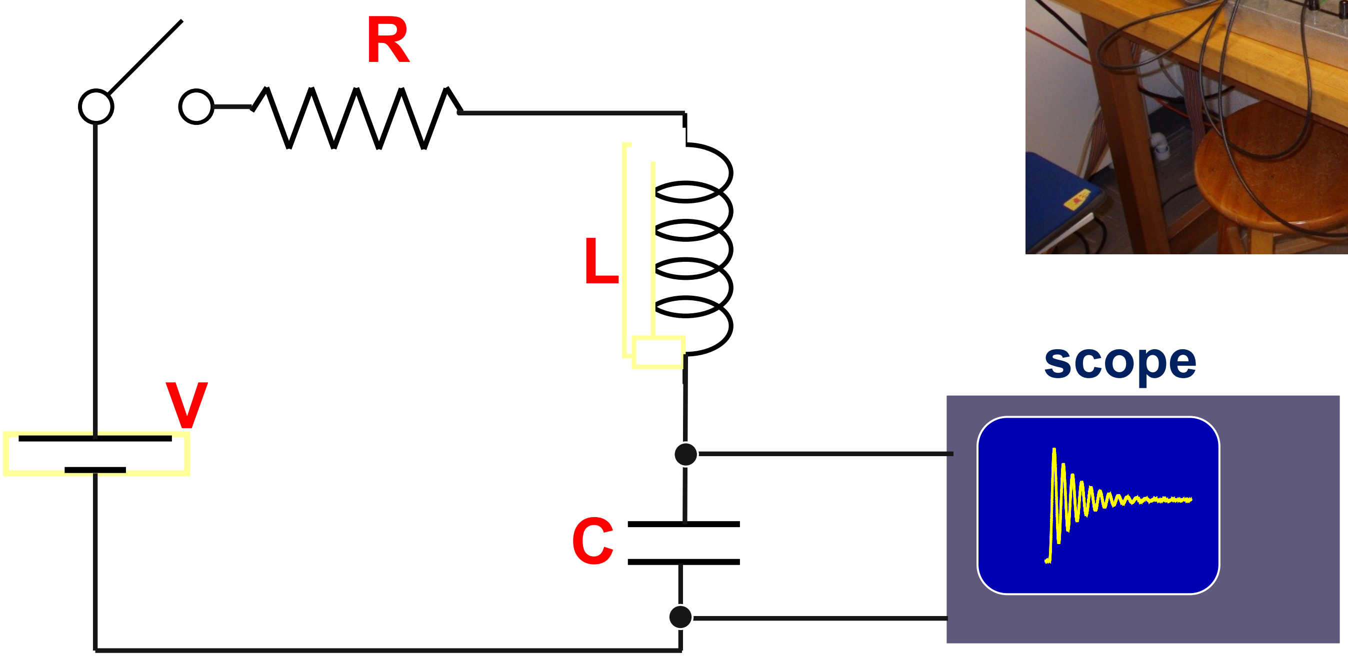
*transient ( physics ) a short-lived oscillation in a system caused by a sudden change of voltage or current or load*

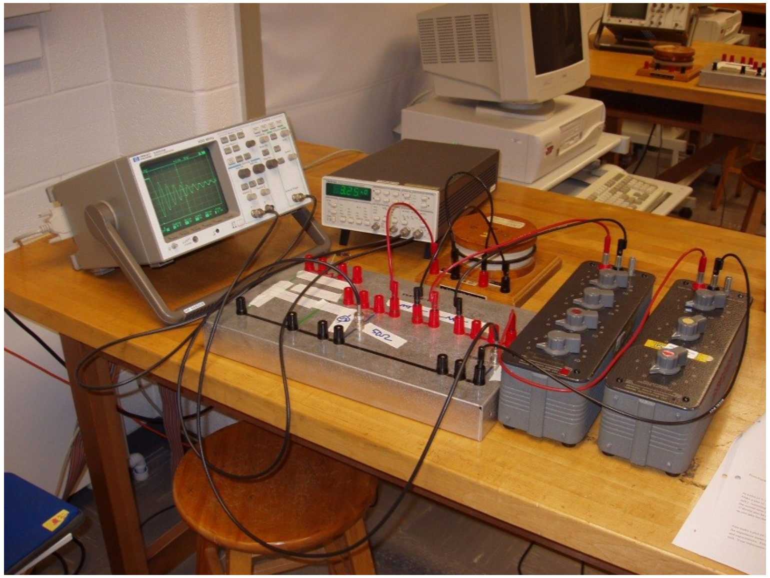
*a transient response or natural response is the response of a system to a change from equilibrium.*



# Transients in RLC circuit.

***Resistance R [Ohm]***
***Capacitance C [uF] (10$^{-6}$F)***
***Inductance L [mH] (10$^{-3}$H)***



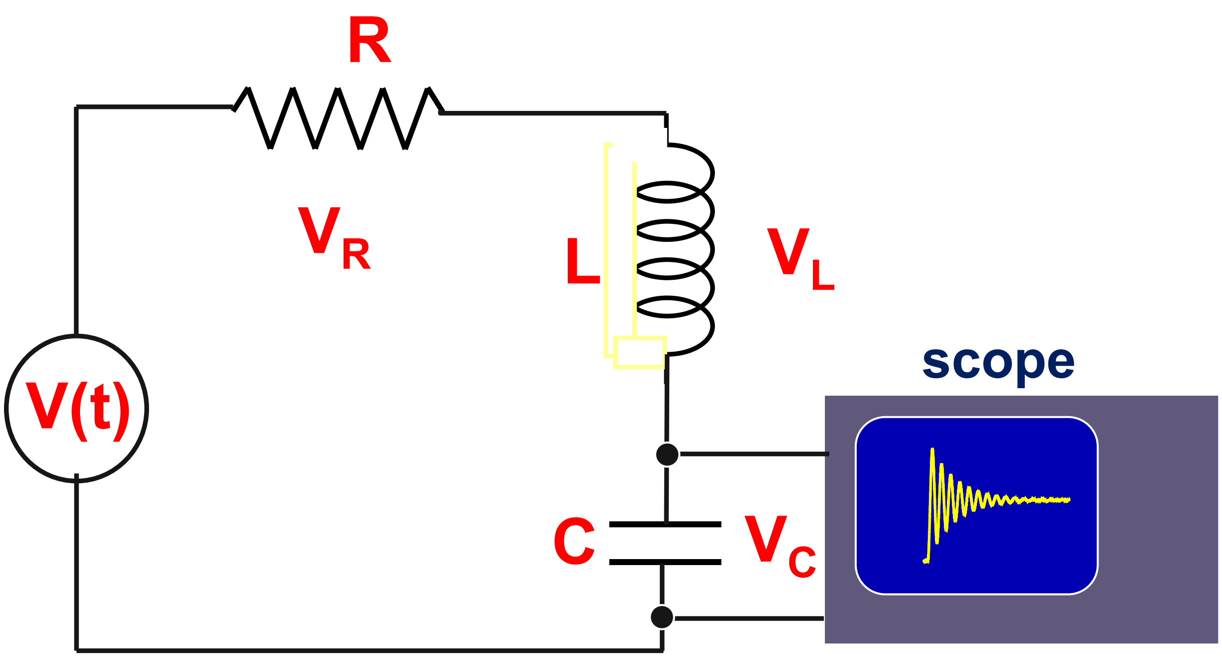



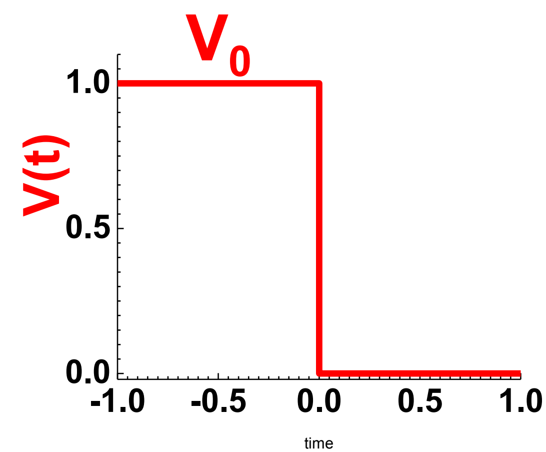
# Transients in RLC circuit.

**According the Kirchhoff's law** $V_R+V_L+V_C=V(t)$

$$L\frac{d^2}{dt^2}q(t)+R\frac{d}{dt}q(t)+\frac{q(t)}{C}=0\ ,\quad \frac{q(t)}{C}=V_0 \qquad (1)$$

*See Lab write-up for details



# Transients in RLC circuit. Three solutions

The solution of this differential equation can be found in the form

$$q(t) = Ae^{st}$$

This will convert (1) in quadratic equation

$$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$$

with solutions:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)} \equiv -a \pm b$$

$$a = \frac{R}{2L} \quad , \quad b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)}$$

$b^2>0$ over-damped solution
$b^2=0$ critically damped solution
$b^2<0$ under-damped solution

# Transients in RLC circuit. Over-damped solution: $b^2>0$

In this case the solution will be aperiodic exponential decay function with no oscillations:

$$q(t) = e^{-at}\left(A_1 e^{bt} + B_1 e^{-bt}\right)$$

$$i(t) = \frac{dq}{dt} = -ae^{-at}(A_1 e^{bt} + B_1 e^{-bt}) + be^{-at}(A_1 e^{bt} - B_1 e^{-bt})$$

# Transients in RLC circuit. Over-damped solution: $b^2>0$

**Taken in account the initial conditions: $q(0)=q_0$ and $i(0)=0$**

$$q(t) = q_0 e^{-at}\left(\cosh bt + \frac{a}{b}\sinh bt\right)$$

$$\xrightarrow[(a-b)t>>1]{} \frac{q_0}{2}\left(1+\frac{a}{b}\right)e^{-(a-b)t}$$

$$i(t) = -\frac{q_0}{2}\left(\frac{a^2-b^2}{b}\right)e^{-(a-b)t}$$

**This is exponential decay function**

# Transients in RLC circuit. Critically-damped solution

$b^2=0$

For this case the general solution can be found as

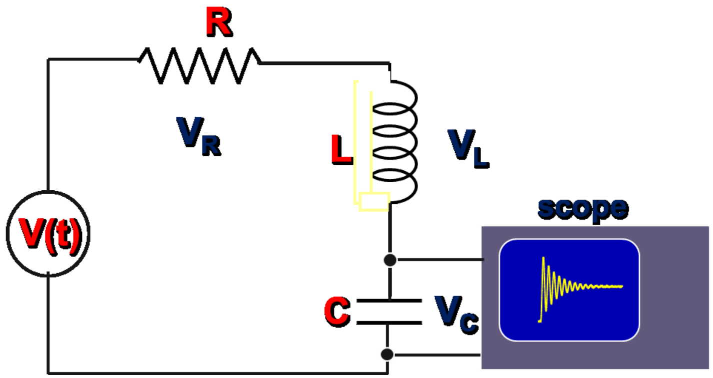
$q(t)=(A_2+B_2t)e^{-at}$. Applying the same initial condition

the current can be written as $i=-a^2q_0te^{-at}$

**Critical damped case shows the fastest decay with no oscillations**

# Transients in RLC circuit. Critically-damped solution. Real data analysis.

**$b^2=0$**



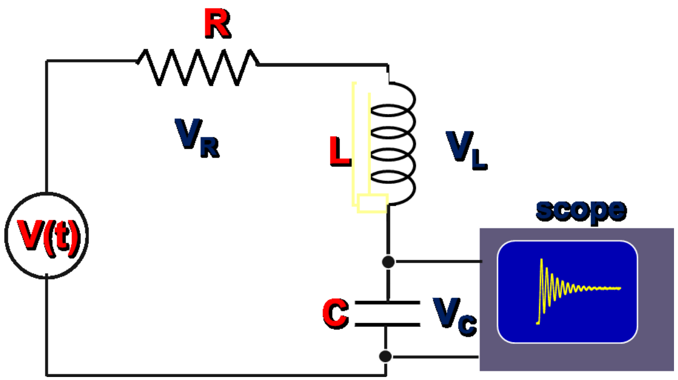

**In this experiment R=300 ohms, C=1μF, L=33.43mH.**

**The output resistance of Wavetek is 50 ohms and resistance of coil was measured as 8.7 ohms, so actual resistance of the network is $R_a$=300+50+8.7=358.7**

**Decay coefficient** $a = \frac{R}{2L} = \frac{358.7}{2*33.43E-3} \approx 5365$

# Transients in RLC circuit. Critically-damped solution. Real data analysis.

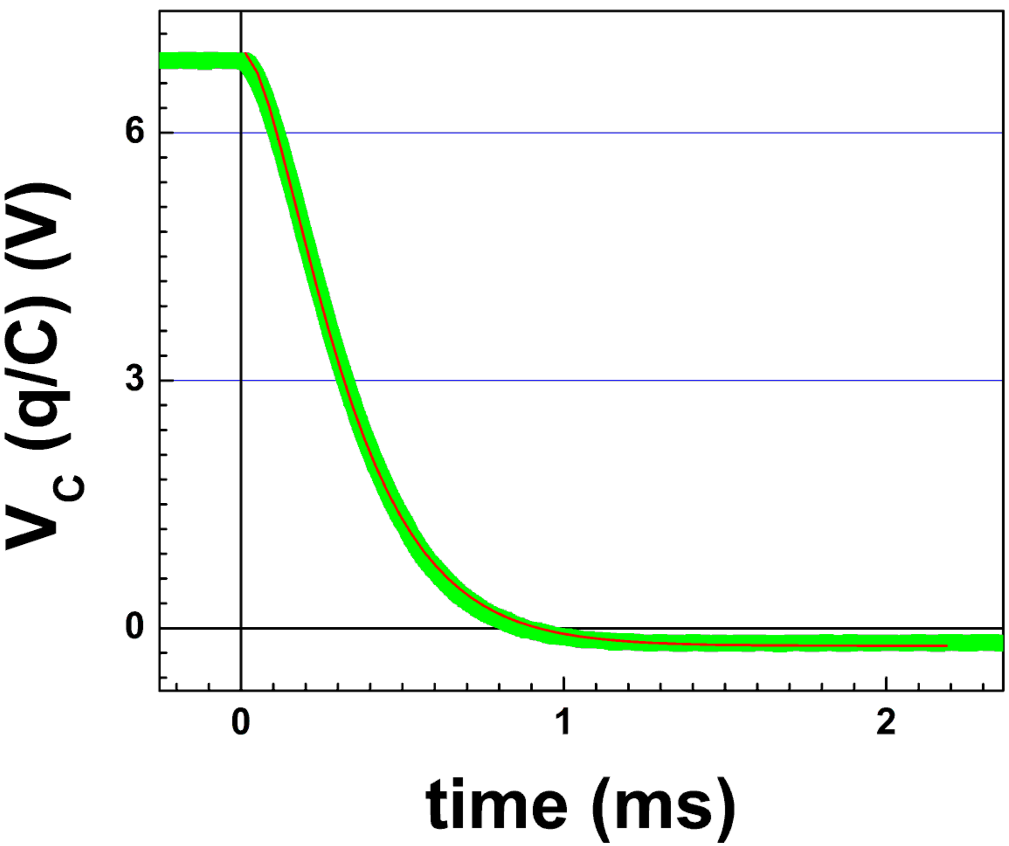
**$b^2=0$**




**Vc ~q, fiiting function: $V_c=V_{co}(1+at)e^{-at}$**

Calculated decay coefficient ~5385,

Obtained from fitting - ~5820.
Possible reason – it is still slightly over damped
Calculated $b^2$ is
$b^2$=2.99e7-2.90e7>0



# Transients in RLC circuit. Under-damped solution.

If **$b^2<0$** we will have oscillating solution. Omitting the details (see Lab write-up) we have the equations for charge and current as:

$$q(t) = q_0 e^{-at}\left(\cos bt + \frac{a}{b}\sin bt\right) = q_0 e^{-at}\sqrt{1+\frac{a^2}{b^2}}\ \sin(bt+\varphi)$$

$$i(t) = q_0 e^{-at}\left(\frac{a^2+b^2}{b}\right)\sin bt$$

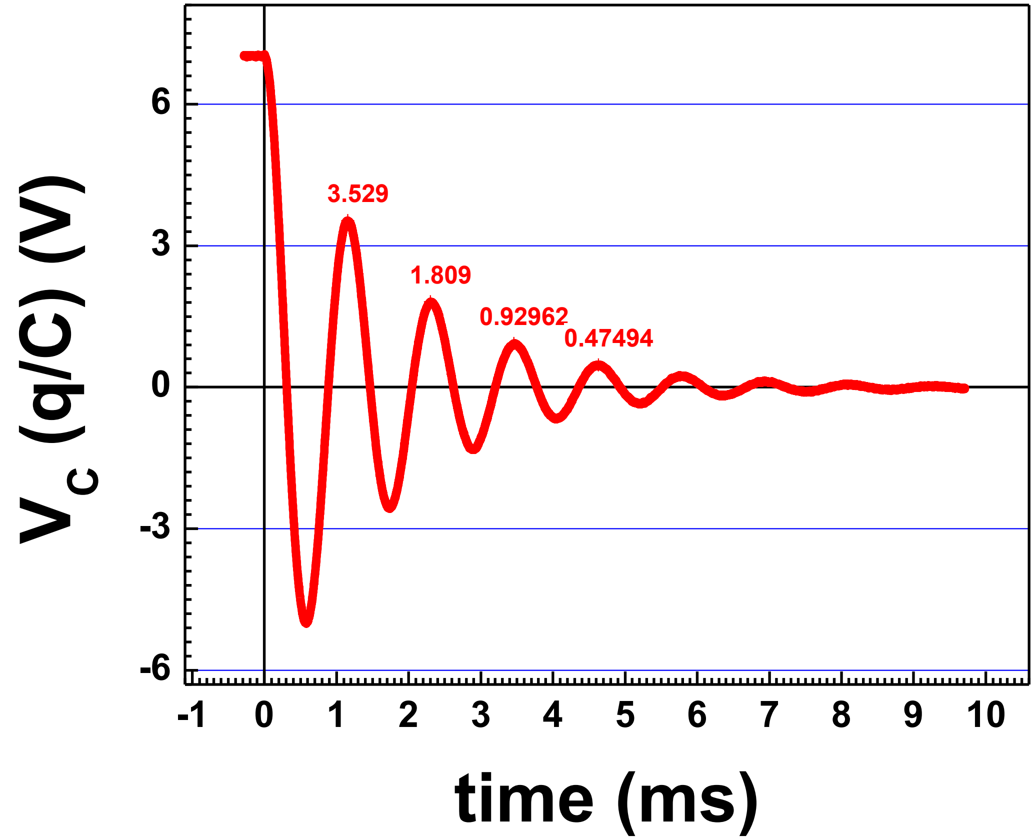
$$a = \frac{R}{2L}\ ,\ b = \sqrt{\left(\frac{R}{2L}\right)^2 - \left(\frac{1}{LC}\right)};f = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC}\right)-\left(\frac{R}{2L}\right)^2}$$

# Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

**Log decrement can be defined as** $\delta = ln\left(\frac{q(t_{max}}{q(t_{max}+T_1}\right) = ln\left(\frac{e^{-at_{max}}}{e^{-at_{max}+T_1}}\right) = aT_1$**, where** $T_1$**=1/f1**

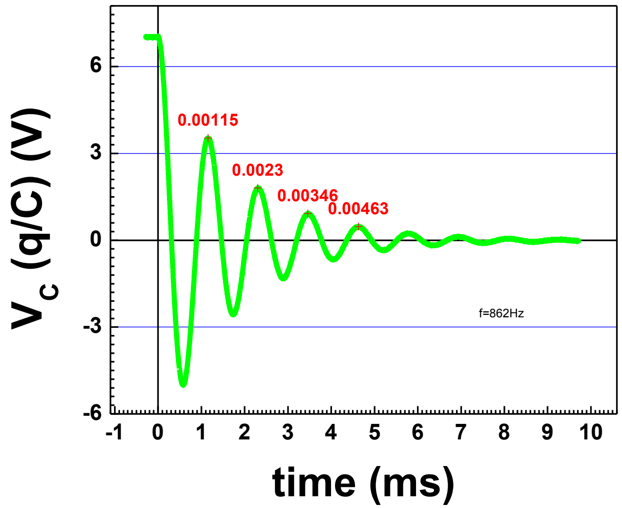
**Quality factor can be defined as** $Q = 2\pi\frac{E}{\Delta E}$**,**

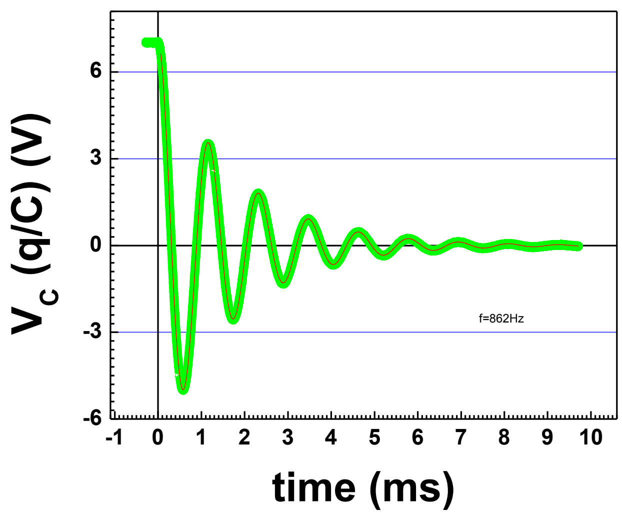
**For RLC** $Q = \frac{\omega_1 L}{R} = \frac{\pi}{\delta}$




**From this plot δ≈0.67**
**Q≈4.7**

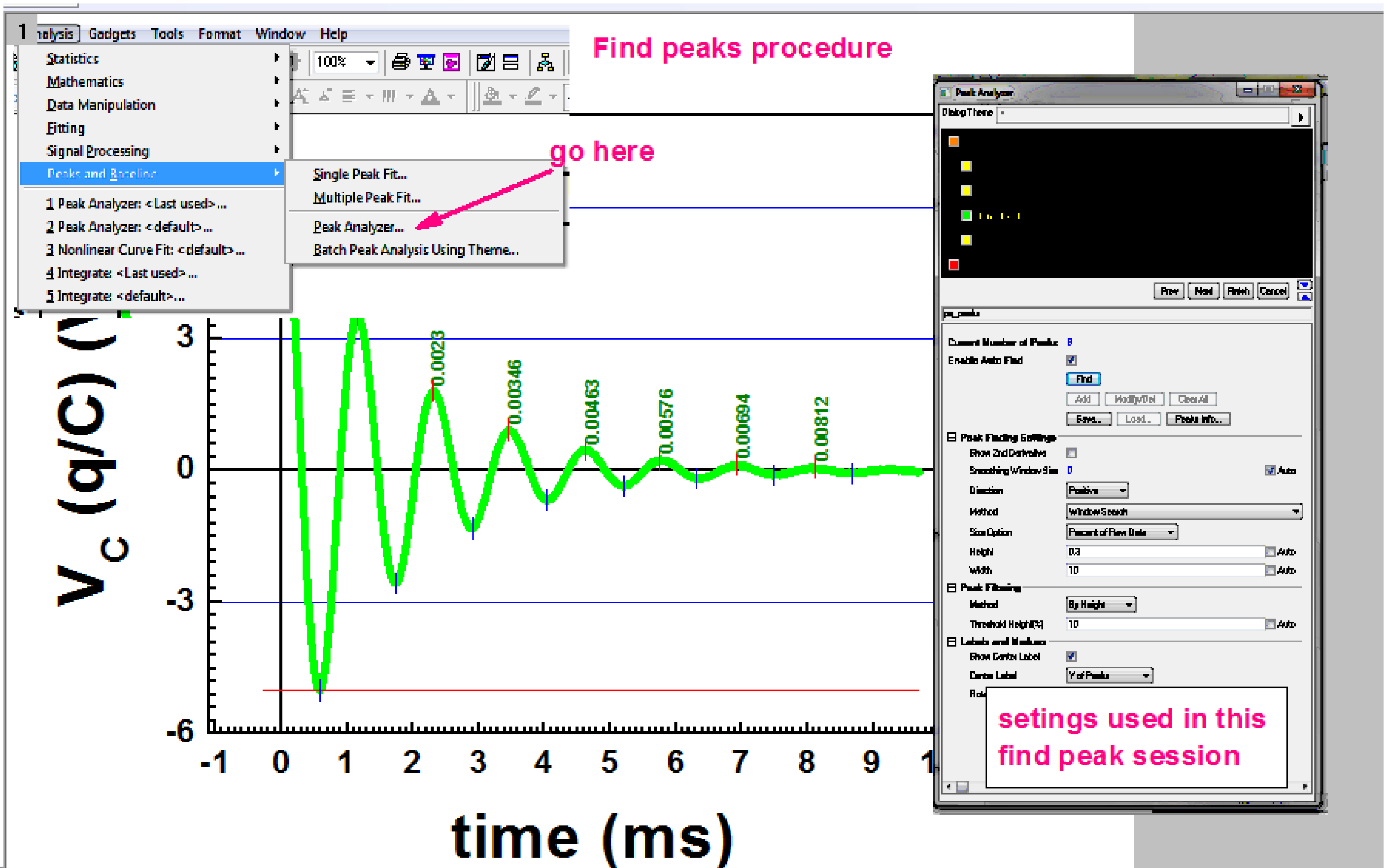
# Transients in RLC circuit. Data analysis. Using Origin software.



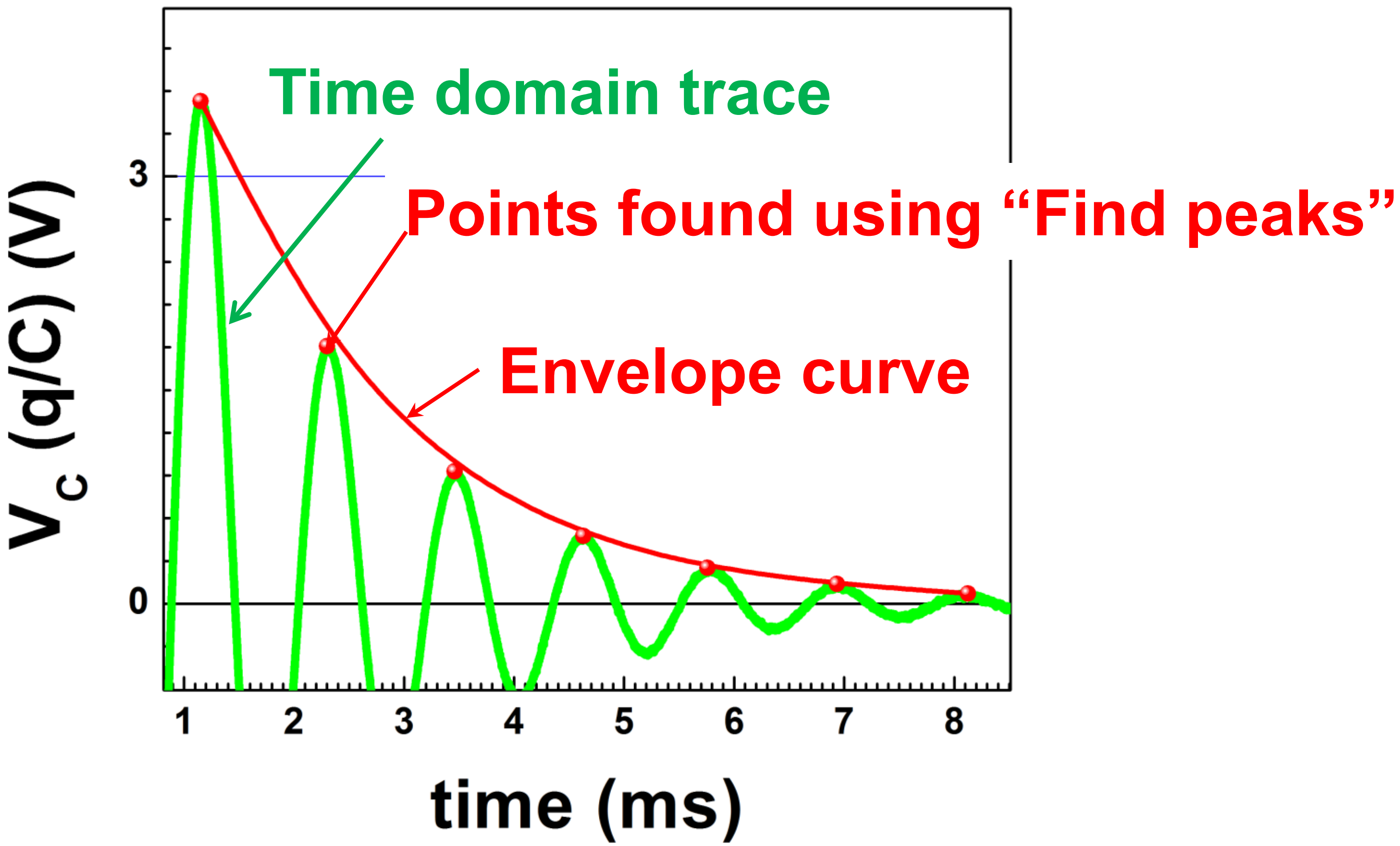




1. **Pick peaks**
2. **Envelope**
3. **Exponential term**
4. **Nonlinear fitting**

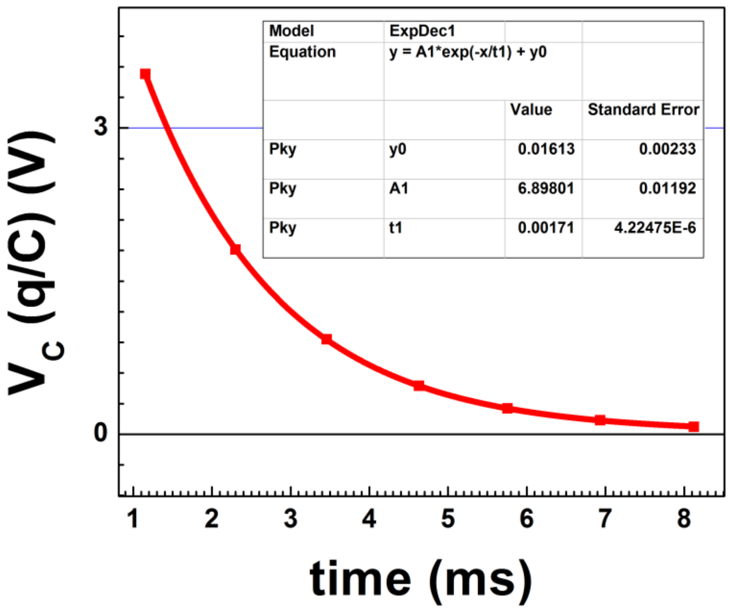
# Transients in RLC circuit. Under-damped solution. Log decrement. Quality factor.

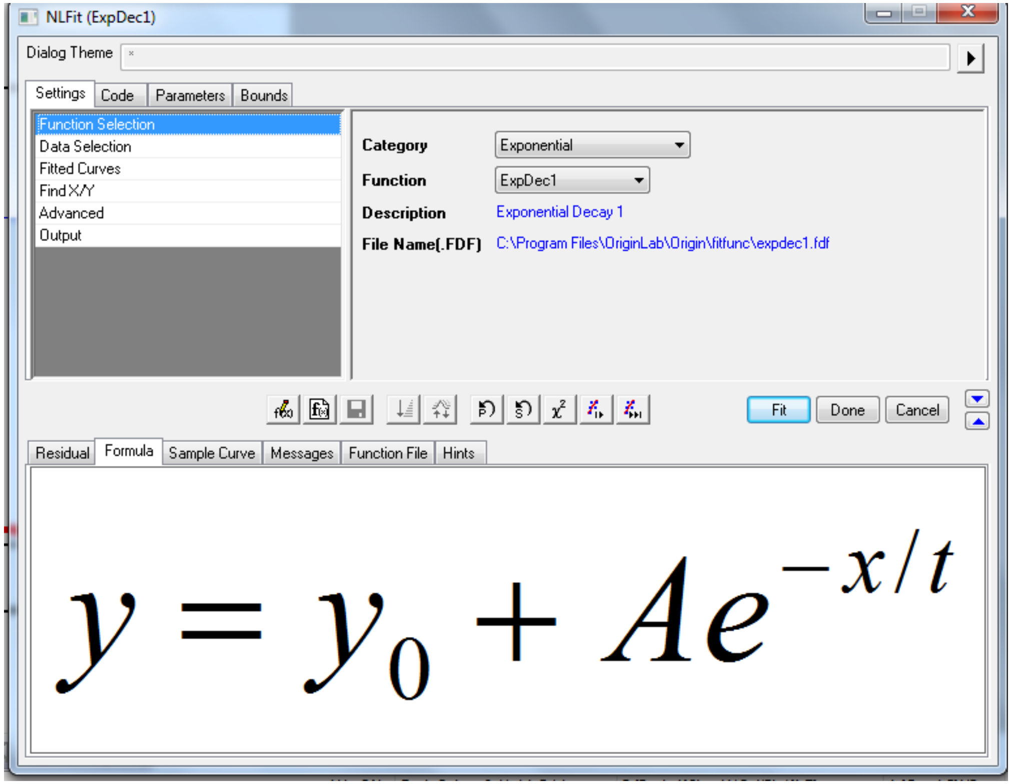


# Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.



# Transients in RLC circuit. Data analysis. Log decrement. Using Origin software. Results.

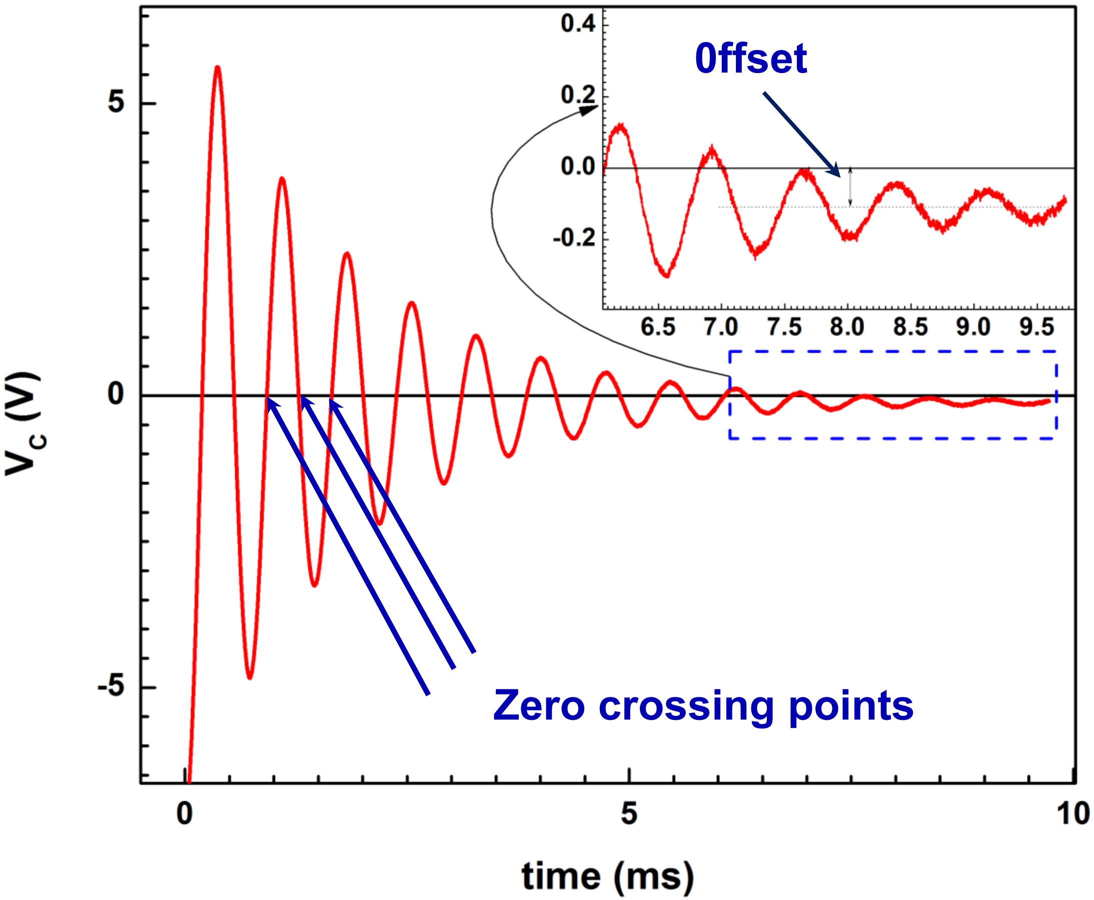



**Fitting the “envelope data” to exponential decay function**

# Transients in RLC circuit. Data analysis. $(1/T)^2$ vs 1/C experiment.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi) + offset$$

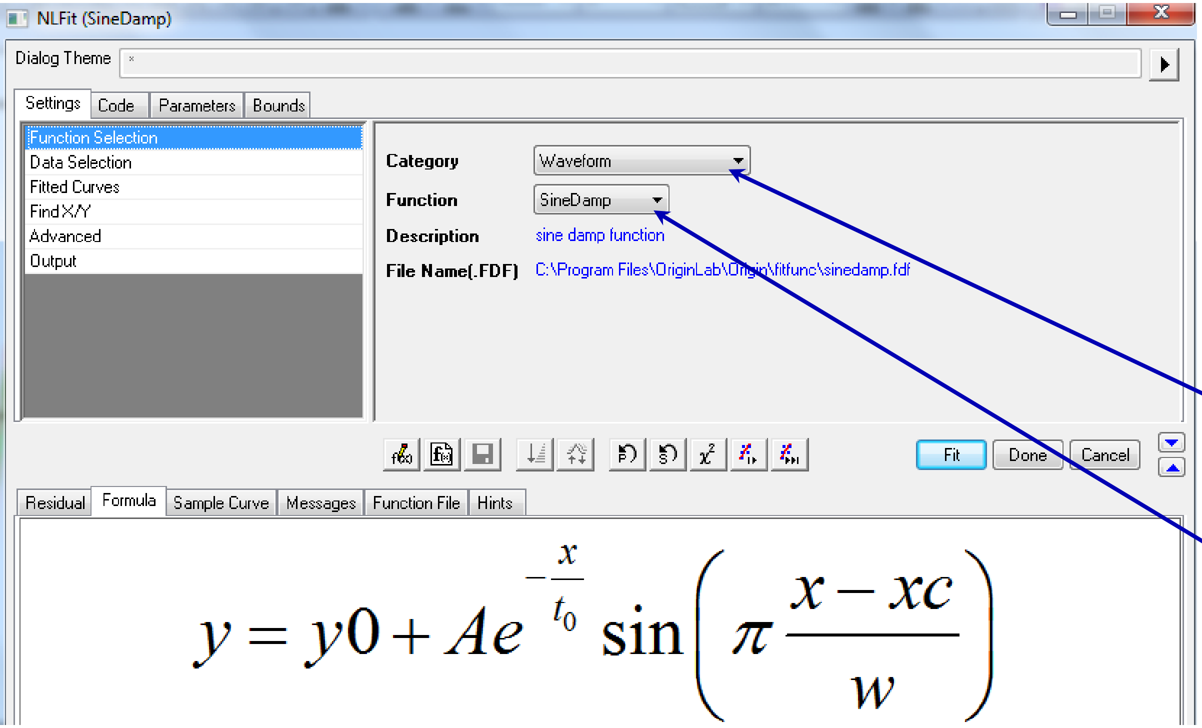



**Manual evaluation of the period of the oscillations**

**Limited accuracy**

**Results can be effected by DC offset**

# Transients in RLC circuit. Data analysis. $(1/T)^2$ vs 1/C experiment. Using Origin software.




$$q(t) = Ae^{-at} \sin(\omega t + \varphi)$$

**Use Origin standard function**

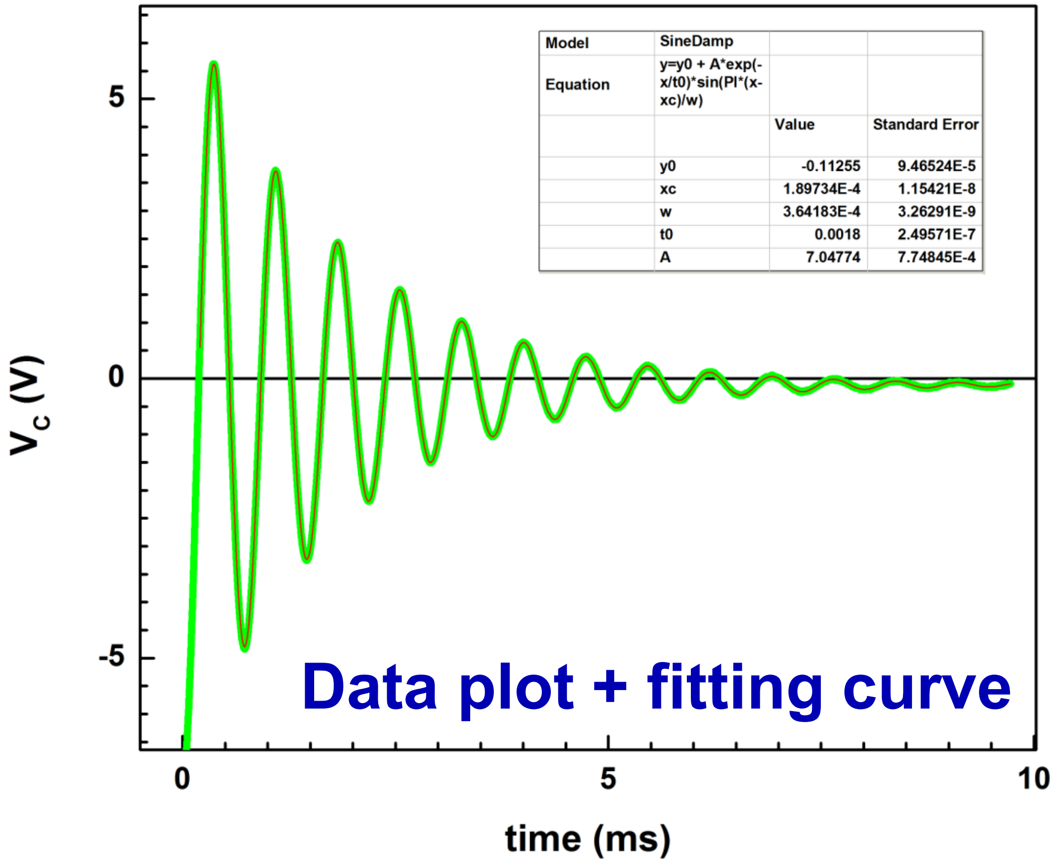
**Category: Waveform**

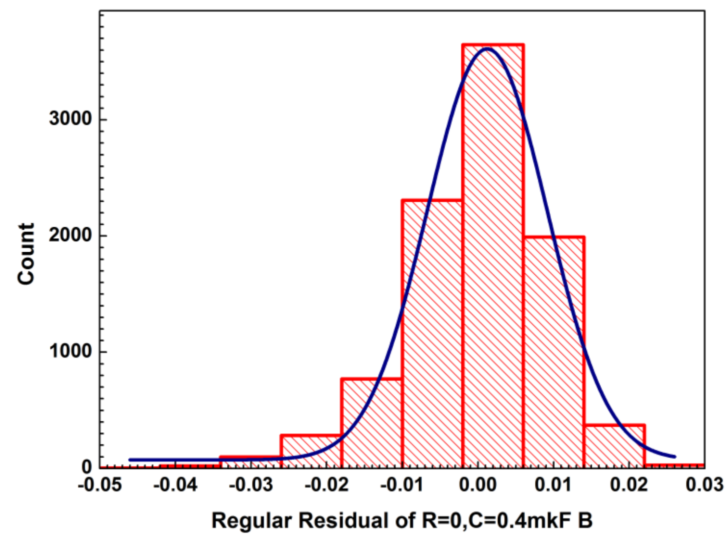
**Function: SineDamp**

**Fitting function ; $y0, A, t_0$ xc, w – fitting parameters**

# Transients in RLC circuit. Data analysis. $(1/T)^2$ vs 1/C experiment. Using Origin software.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$

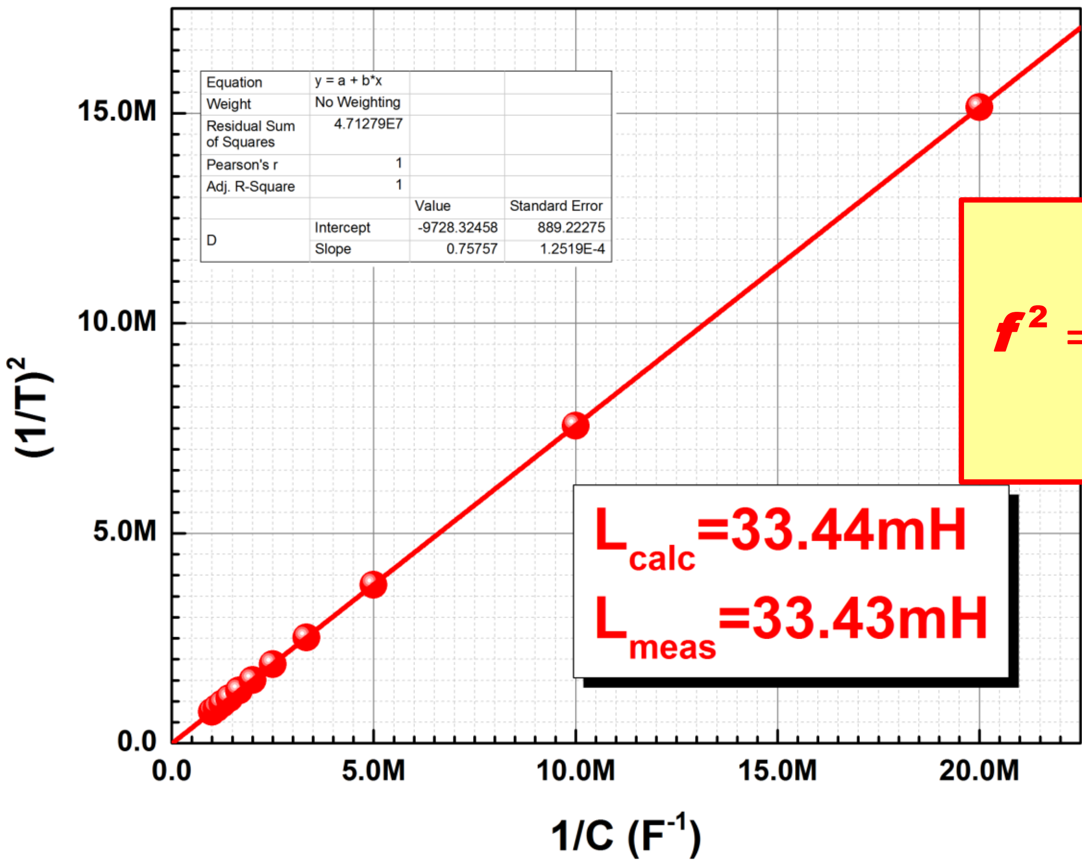



**Residuals - criteria of quality of fitting**

# Transients in RLC circuit. Data analysis. $(1/T)^2$ vs 1/C experiment. Using Origin software.

$$q(t) = Ae^{-at}\sin(\omega t + \varphi)$$



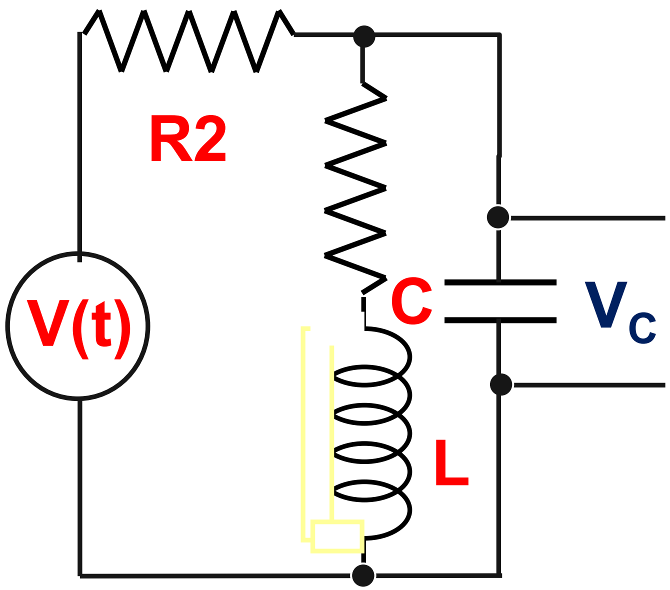

$$f^2 = \left(\frac{1}{T}\right)^2 = \frac{1}{2\pi}\left(\left(\frac{1}{LC}\right) - \left(\frac{R}{2L}\right)^2\right)$$

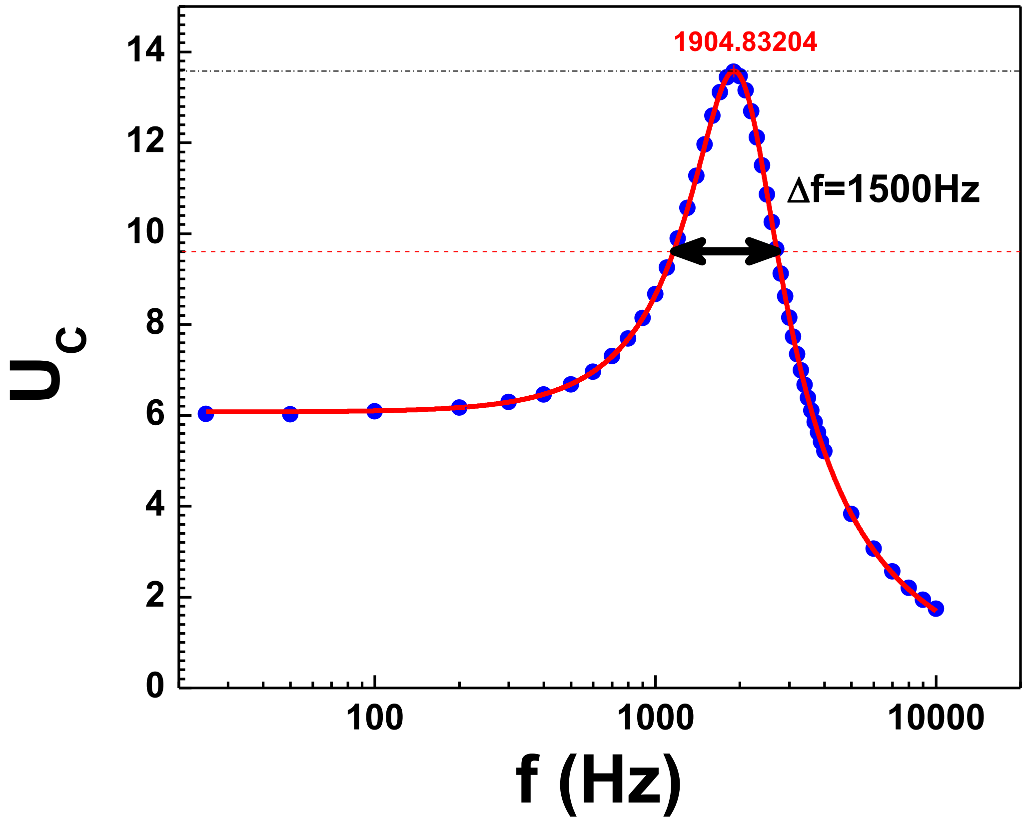
$L_{calc}$=33.44mH

$L_{meas}$=33.43mH

**Final results**

# Resonance in RLC circuit.

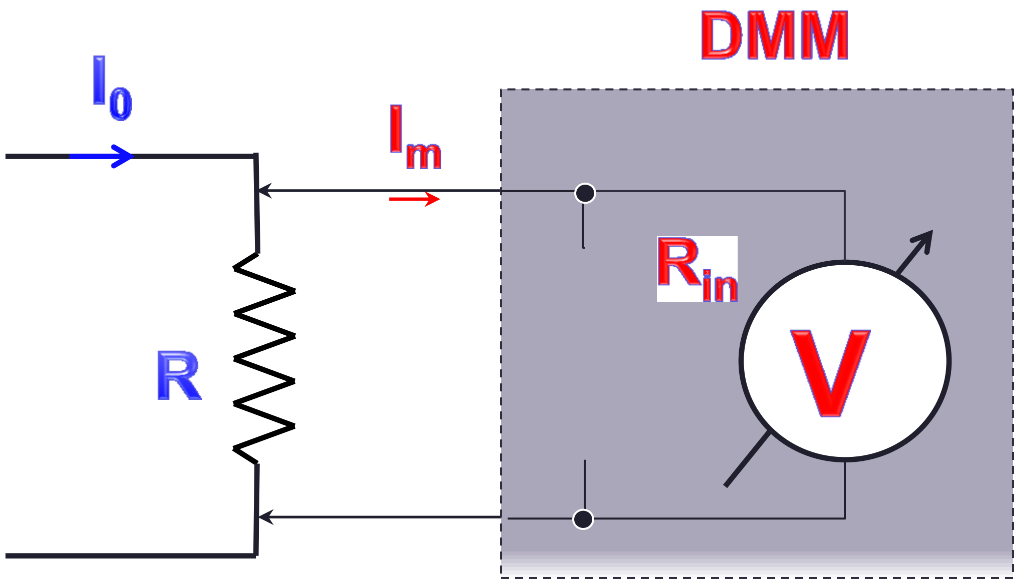



$$Q = \frac{f}{\Delta f} = \frac{1904}{1500} = 1.26$$

# Input resistance. Measuring the voltage.

**One important parameter for any electrical measuring equipment (DMM, scope, amplifier) is its input resistance**




***Ideal case:***

$R_{in}=\infty$;

$I_m=0$;

$V=R*I_0$

***Real situation*:**

$R_{in}\neq\infty$;

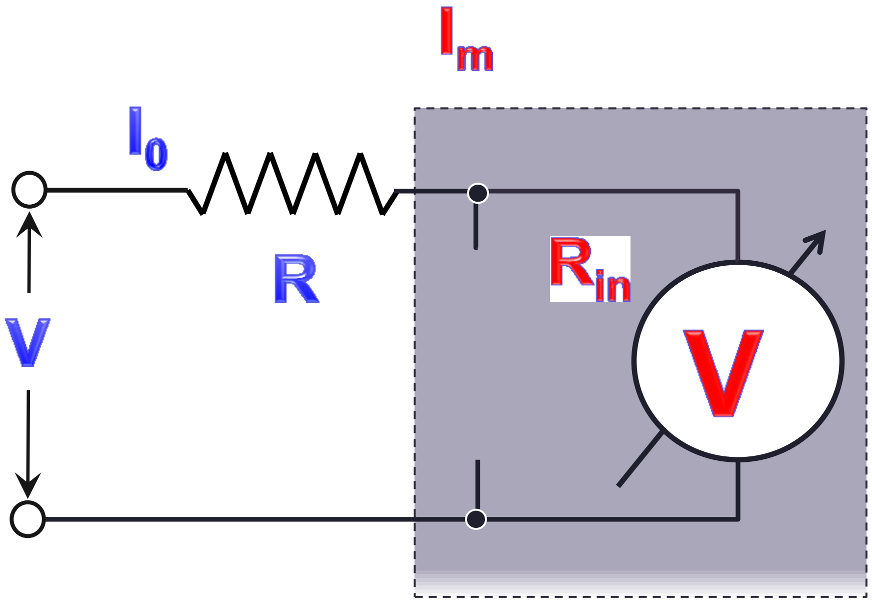
$$I_m=I_0\left(\frac{\frac{R}{R_{in}}}{1+\frac{R}{R_{in}}}\right);$$

$$V=I_0\left(\frac{R}{1+\frac{R}{R_{in}}}\right)$$

**For correct voltage measurements it necessary to have $R_{in}>>R$**

# Input resistance. Measuring the current.

**The same about current measurement, but for ampere meter the requirement for $R_{in}$ is much different.**




*Ideal case:*

$I_0 = \frac{V}{R}$ (expected value)

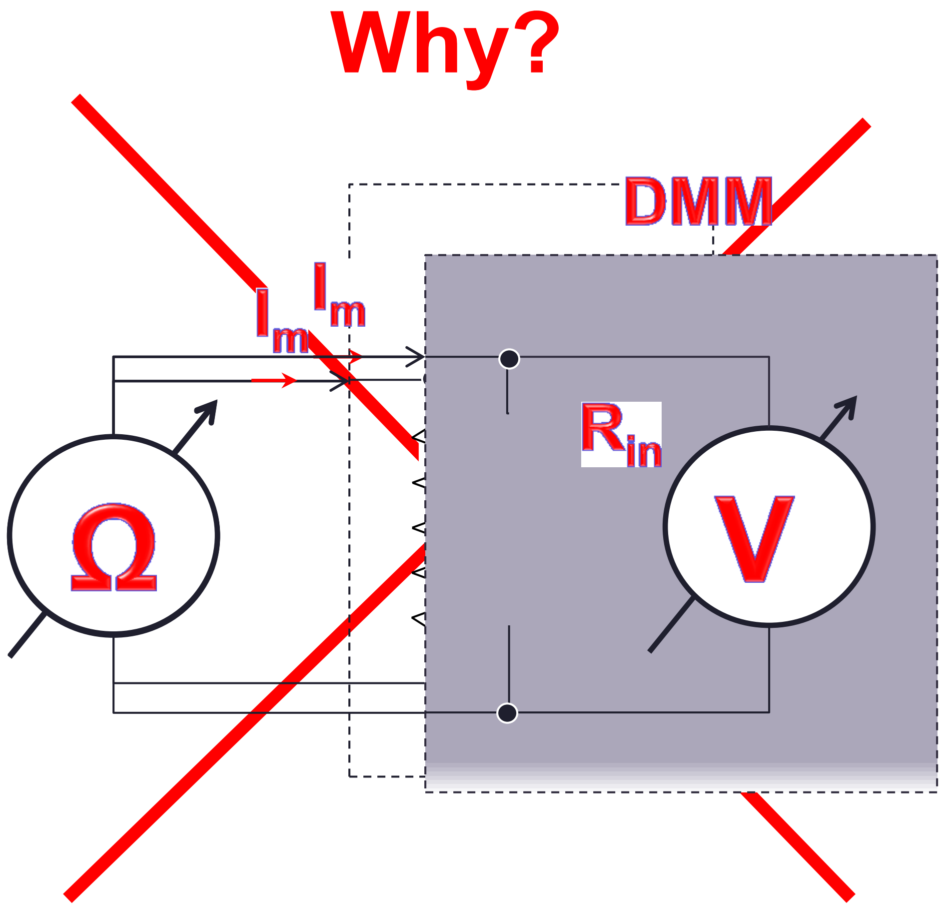
***Real situation***:

$$I = \frac{V}{R + R_{in}}$$

$I \approx I_0$ if $R_{in} \to 0$

**For correct current measurements it necessary to have $R_{in} << R$**

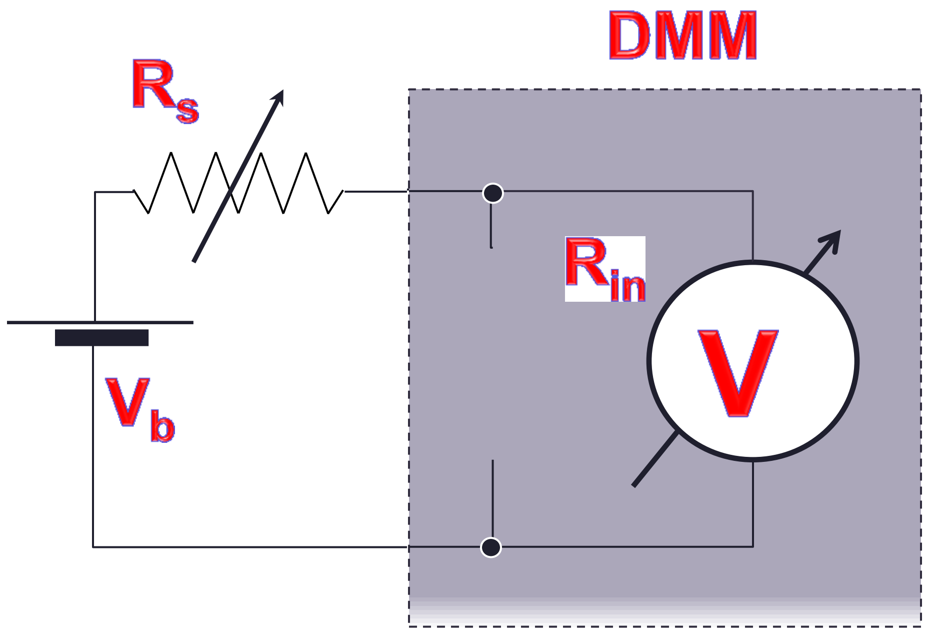
# Measuring of the input resistance of the voltmeter.




**The current applied by ohmmeter to the input of amplifier or DMM can destroy the electronics**

# Measuring the input resistance of voltmeter.

## Correct idea



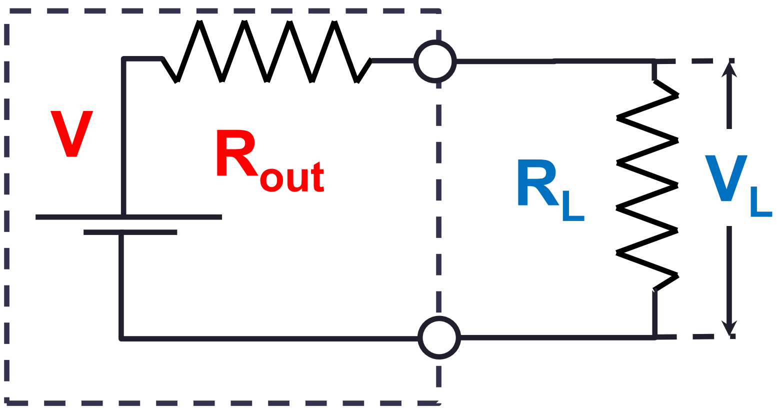

**Ignoring battery output resistance $V_b$ we can measure if $R_s=0$. Next we have to vary $R_s$ until DMM reading becomes equal 0.5 $V_b$**

$$V = 0.5V_b = \frac{R_{in}}{R_s + R_{in}} V_b$$

$$\Rightarrow R_s = R_{in}$$

# Output resistance. Voltage source.

***Equivalent circuit of voltage source***




**Voltage on the load:**

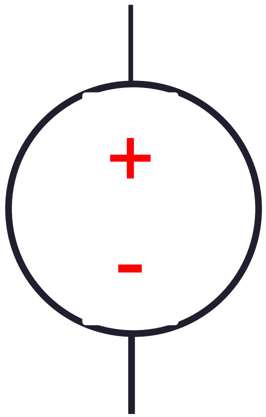
$$V_L = V \times \frac{R_L}{R_L + R_{out}}$$

$$V_L \approx V \text{ if } R_{out} << R_L$$

**Power on the load:**

$$P = \frac{V_L^2}{R_L} = \frac{V^2 R_L}{(R_L + R_{out})^2}$$

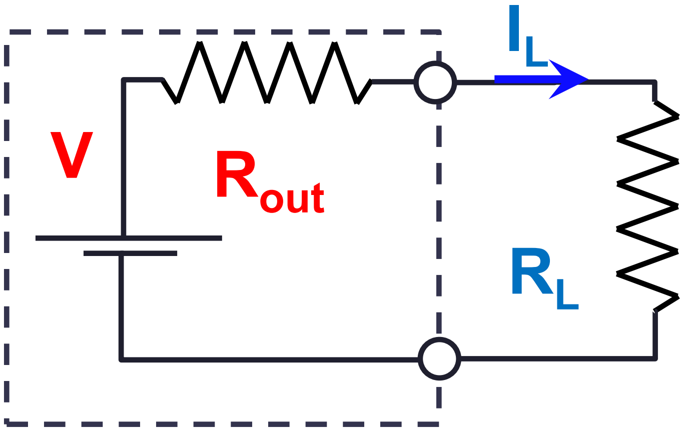
$$P = P_{max} = \frac{V^2}{4R_{out}} \text{ if } R_L = R_{out}$$




***Voltage source circuit diagram symbol***

# Output resistance. Current Source.

*Equivalent circuit of current source*



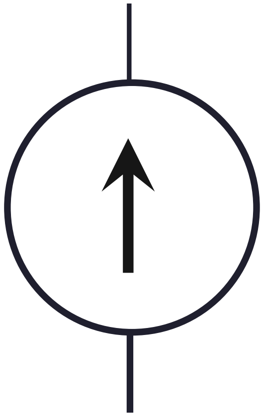

**Ideal current source – current in the load should not depend on the load resistance.**

$$I_L = \frac{V}{R_L + R_{out}} = \frac{I_0}{1 + \frac{R_L}{R_{out}}},$$

**where**

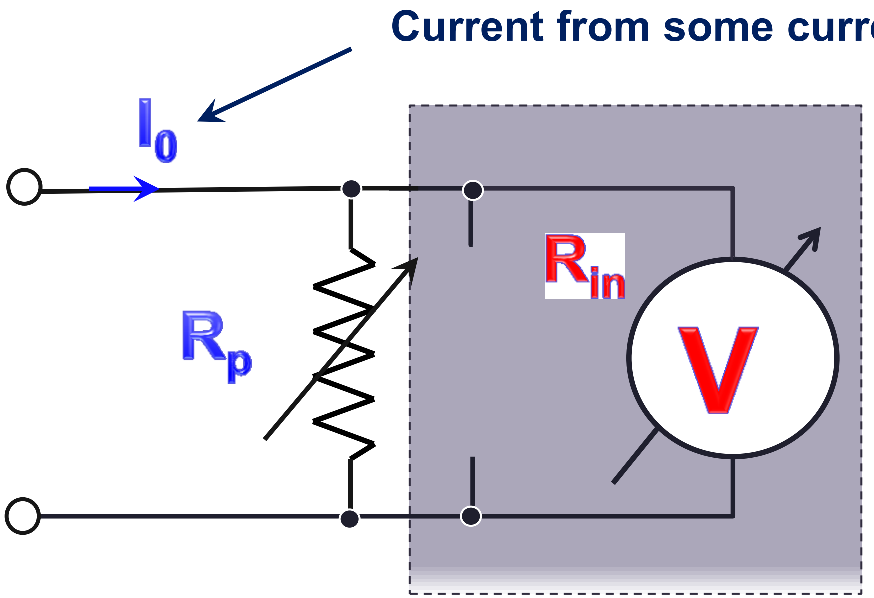
$$I_0 = \frac{V}{R_{out}}$$

**So, for ideal current source $R_{out} \to \infty$**



*Current source circuit diagram symbol*

# Measuring of the Input resistance of the current meter.

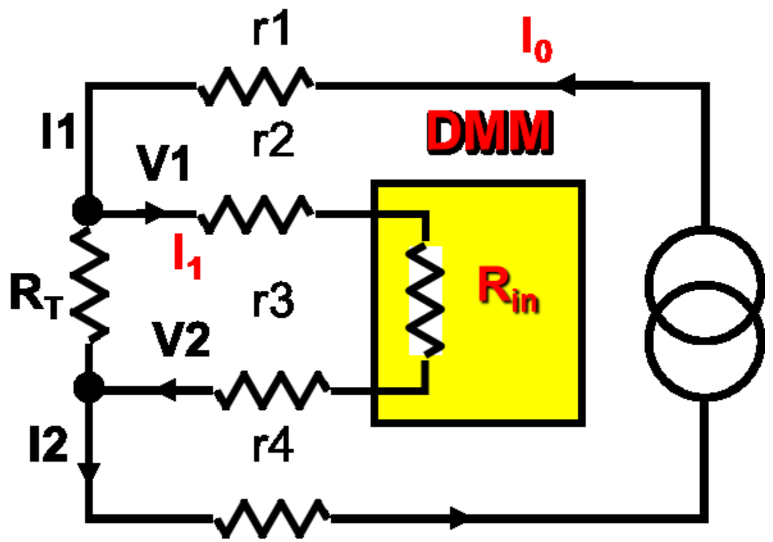


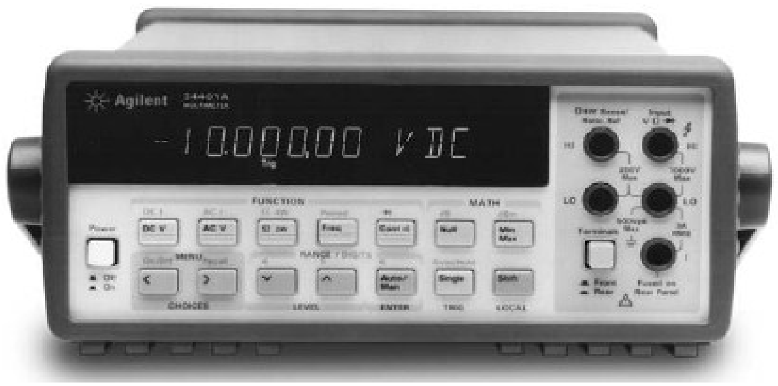

***For measurement of the input resistance of the current meter we can use the current divider .***

***Varying the resistance of the parallel resistor $R_p$ we have to find the value of $R_p$ corresponding the reading of current equal $I_0/2$. In this case***

$$R_{in}=Rp$$

# Measuring of voltage, resistance. Four probe technique



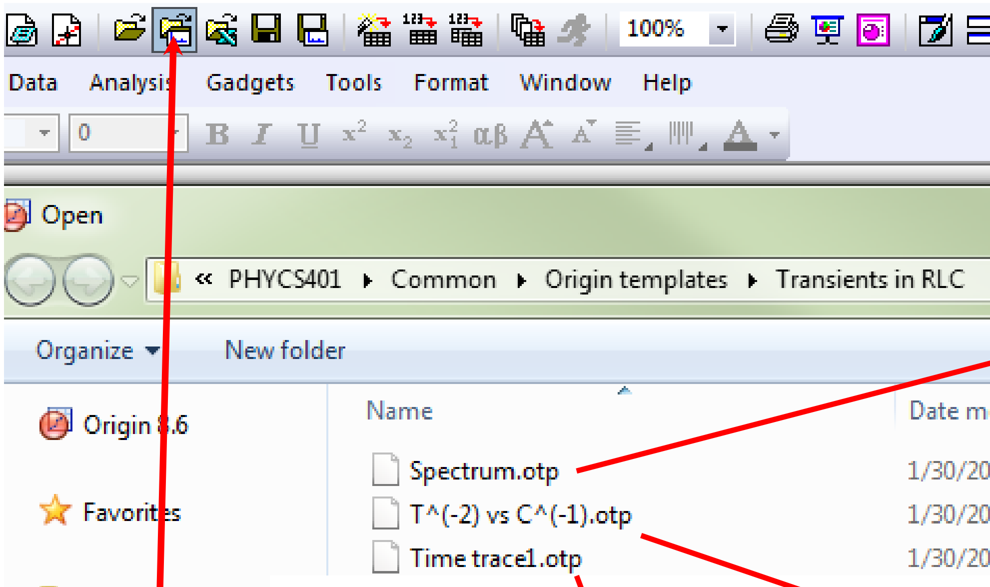
**Most of DMM's have four probe option for resistance measurements.**

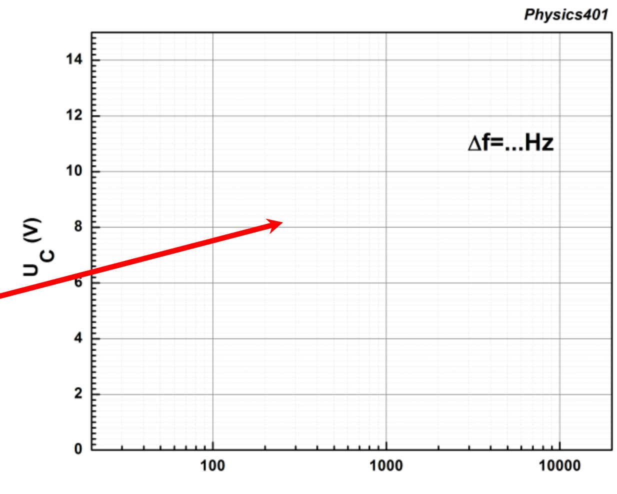
If the sensor is mounted in cryostat the overall leads resistance could reach a couple of ohms. This will In case of RTD100 the resistance at 0°C is 100Ω give an error of a couple percent!

$$I1 = Io * R_T / (r2 + r3 + R_{in}) \sim 0$$

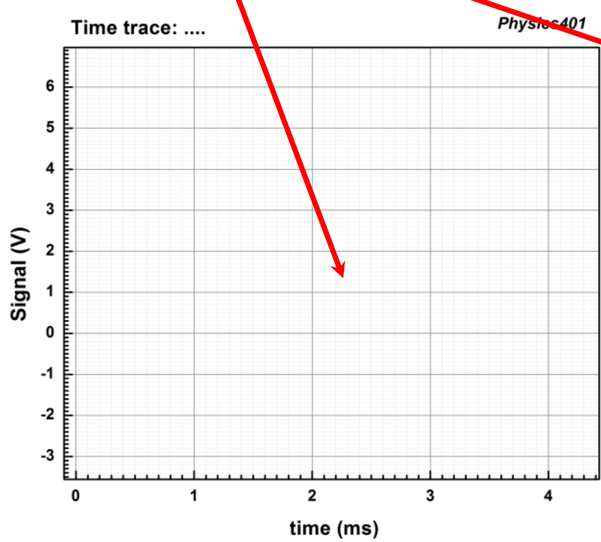
$$R_{in} \sim 10^{10}\Omega$$

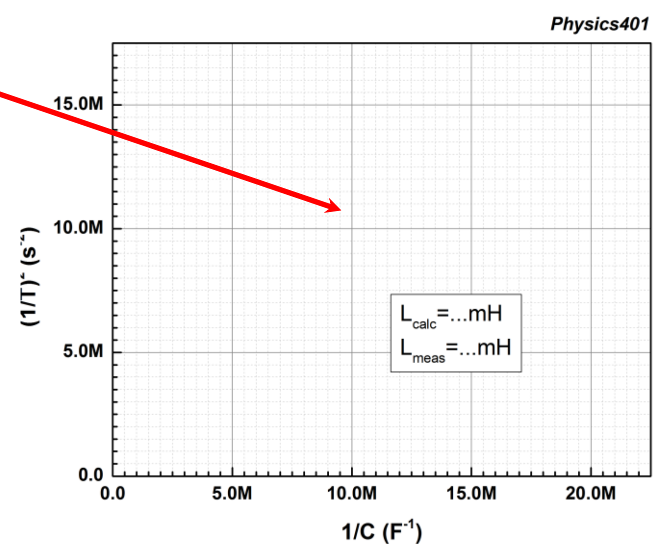
# Origin templates for this week Lab.



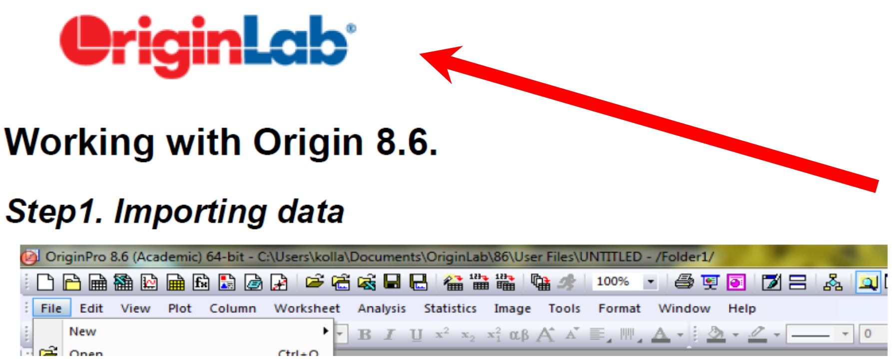




***Open template* button**



# Origin manuals

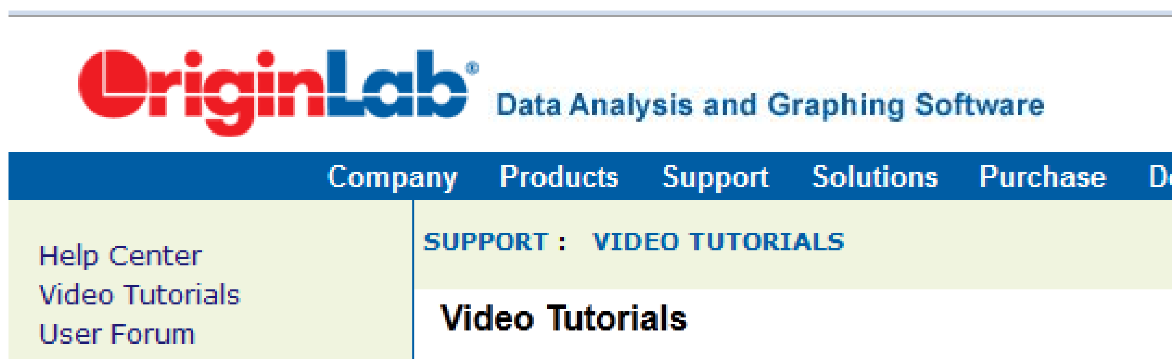



**Very short and simple manual which covers only main general operations with Origin. Document located on server (\\PhyapIportal\PHYCS401\Common\Origin manuals) and there is a link from P401 WEB page**

**There are (\\PhyapIportal\PHYCS401\Common\Origin manuals) also manuals from OriginLab.**

**Do not forget about Origin Help**

**Video Tutorials at the site of the company**




***http://www.originlab.com/index.aspx?go=SUPPORT/VideoTutorials***